$$y' = -\frac{2x+y-2}{2y+x}$$

$$\frac{dy}{dx} = -\frac{2x+y-2}{2y+x}$$

$$(2x+y-2)\,dx + (2y+x)\,dy = 0$$

$$\frac{\partial P}{\partial y} = 1 \qquad \frac{\partial Q}{\partial x} = 1 \qquad \text{Exacta}$$

$$\int (2x+y-2)\,dx = \frac{4x^2}{2} + xy - 2x + C(y) = C$$

$$= 2x^2 - 2x + xy + C(y)$$

$$2x^2 - 2x + xy$$

$$4x - 2 + y + C'(y) = 2y + x$$

$$C'(y) = 2y + x - 4x - y + 2$$

$$= y - 3x + 2$$

$$C(y) = \frac{y^2}{2} - 3xy + 2y$$

$$2x^2 - 2x + xy + \frac{y^2}{2} - 3xy + 2y = C$$

$$2x^2 - 2xy + \frac{y^2}{2} - 2x + 2y = C$$

$$x^2 - xy + \frac{y^2}{4} - x + y = C$$

$$x \ln x \, dy + (y - \ln x)\, dx = 0$$

$$x \ln x \frac{dy}{dx} + y - \ln x = 0$$

$$y' + \frac{y}{x \ln x} - \frac{\ln x}{x \ln x} = 0$$

$$y' + \frac{1}{x \ln x} y - \frac{1}{x} = 0$$

$$y' + \frac{1}{x \ln x} y = \frac{1}{x}$$ lineal 1er orden

1°) $\int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln(\ln x)$

2°) $e^{\ln(\ln x)} = \ln x$

3°) $\int \ln x \cdot \frac{1}{x} dx = \frac{\ln^2 x}{2}$

$$y \ln x = \frac{\ln^2 x}{2} + \ln C$$

$$\ln x^y = \frac{\ln^2 x}{2} + \ln C$$

$$y' + e^x y = e^x \qquad \text{lineal } 1^{er} \text{ orden}$$

$$1^o)\ \int e^x\,dx = e^x$$

$$2^o)\ e^{e^x}$$

$$3^o)\ \int e^{e^x}\cdot e^x\,dx = e^{e^x}$$

$$e^{e^x} y = e^{e^x} + e^c$$

$$y = 1 + e^{c - e^x}$$

$$xy\,y' = x^2 + y^2$$

$$y' = \frac{x}{y} + \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x} \qquad \text{Homogenea grado cero}$$

$$(x^2+y^2)\,dx - x y\,dy = 0$$

$$\frac{\partial P}{\partial y} = 2y \qquad \frac{\partial Q}{\partial x} = -y$$

$$\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = -\frac{1}{xy}\left(2y+y\right) = \frac{3y}{-xy} = -\frac{3}{x} = f(x)$$

factor integrante

$$\mu = e^{\int -\frac{3}{x}dx} = e^{-3\ln x} = e^{\ln x^{-3}} = \frac{1}{x^3}$$

$$\left(\frac{1}{x} + \frac{y^2}{x^3}\right)dx - \frac{y}{x^2}\,dy = 0 \qquad \text{exacta}$$

$$\int\left(\frac{1}{x} + \frac{y^2}{x^3}\right)dx = \ln x - \frac{y^2}{2x^2} + C(y)$$

$$-\frac{2y}{2x^2} + C'(y) = -\frac{y}{x^2}$$

$$C'(y) = 0$$

$$C(y) = k$$

$$\ln x - \frac{y^2}{2x^2} + k = C$$

$$\ln x - \frac{y^2}{2x^2} = C$$

$$2x^2\ln x = C + y^2.$$

Poniendola en la forma

$$y' = \frac{x}{y} + \frac{y}{x}$$

$$y' - \frac{1}{x}y = xy^{-1} \qquad \text{Bernouilli}$$

$$u = y^{1-n} = y^2; \quad y = u^{\frac{1}{2}} \quad y' = \frac{1}{2}u^{-\frac{1}{2}}u'.$$

$$\frac{1}{2}\,\frac{1}{u^{\frac{1}{2}}}u' - \frac{1}{x}u^{\frac{1}{2}} = xu^{-\frac{1}{2}} \qquad \text{multiplicando por } 2u^{1/2}$$

$$u' - \frac{2}{x}u = 2x$$

1°) $\int -\frac{2}{x}\,dx = -2\ln x = \ln\frac{1}{x^2}$

2°) $e^{\ln\frac{1}{x^2}} = \frac{1}{x^2}$

3°) $\int \frac{1}{x^2}\,2x\,dx = \int \frac{2}{x}\,dx = 2\ln x$

$$\frac{y^2}{x^2} = 2\ln x - 2C$$

$$\frac{y^2}{2x^2} = \ln x - C$$

$$y^2 + C = 2x^2\ln x$$

$$(3x^2 + 4y\,\mathrm{sen}\,x\cos x)\,dx + (2\,\mathrm{sen}^2x + 3y^2)\,dy = 0$$

$\frac{\partial P}{\partial y} = 4\,\mathrm{sen}\,x\cos x$ $\qquad$ $\frac{\partial Q}{\partial x} = 4\,\mathrm{sen}\,x\cos x$ $\qquad$ Exacta

1°) $\int (2\,\mathrm{sen}^2x + 3y^2)\,dy = 2y\,\mathrm{sen}^2x + \int 3y^2\,dy = 2y\,\mathrm{sen}^2x + y^3 + C(x)$

2°) $4y\,\mathrm{sen}\,x\cos x + C'(x) = 4y\,\mathrm{sen}\,x\cos x + 3x^2$

$$C'(x) = 3x^2$$

$$C(x) = x^3$$

$$2y\,\mathrm{sen}^2x + y^3 + x^3 = C$$

$y' = -\dfrac{x+y}{x}$ homogenea, lineal y exacta

$y' + \frac{1}{x} y = -1$ lineal 1er orden

1°) $\int \frac{1}{x}\,dx = \ln x$

2°) $e^{\ln x} = x$

3°) $\int -x\,dx = -\frac{x^2}{2}$

$$xy = -\frac{x^2}{2} + C$$

$$y = \frac{C}{x} - \frac{x}{2}$$

$y\,dx + (2\sqrt{xy} - x)\,dy = 0$ Homogenea $m=1$

$\left.\begin{array}{l} y = ux \\ dy = u\,dx + x\,du \end{array}\right\} \; ux\,dx + (2\sqrt{xux} - x)(u\,dx + x\,du) = 0$

$$\cancel{u\,dx} + 2\sqrt{u}\,u\,dx + 2x\sqrt{u}\,du - \cancel{u\,dx} - x\,du = 0$$

$$2\sqrt{u}\,u\,dx = (x - 2x\sqrt{u})\,du$$

$$2\sqrt{u}\cdot u\,dx = x(1-2\sqrt{u})\,du$$

$$\int \frac{dx}{x} = \int \frac{1-2\sqrt{u}}{2\sqrt{u}\,u}\,du$$

$$\ln x = \int \left(\frac{1}{2} u^{\frac{3}{2}} - \frac{1}{u}\right) du$$

$$\ln x = \frac{1}{2}\frac{u^{-1/2}}{-1/2} - \ln u - \ln C$$

$$\ln x + \ln u + \ln C = -\frac{1}{\sqrt{u}}$$

$$\ln Cxu = -\sqrt{\frac{1}{u}}$$

$$\ln Cy = -\sqrt{\frac{x}{y}}$$

$$Cy = e^{-\sqrt{\frac{x}{y}}}$$

IMPORTANTE

Resolver $y^{(IV)} - 6y''' + 17y'' - 20y' + 8y = 0$

$$\alpha^4 - 6\alpha^3 + 17\alpha^2 - 20\alpha + 8 = 0$$

	1	-6	17	-20	8
1		1	-5	12	-8
	1	-5	12	-8	0
1		1	-4	8	
	1	-4	8	0	

$$\alpha^2 - 4\alpha + 8 = 0$$

$$\alpha = 2 \pm \sqrt{4-8} = 2 \pm \sqrt{-4} = 2 \pm 2i$$

$$c_1 e + c_2 x e + (c_3 \cos 2x + c_4 \operatorname{sen} 2x) e^{2x}$$