(22) Resolver $\frac{dy}{dx} = \frac{y+x}{y-x}$

$(y+x)\,dx = (y-x)\,dy$ Homogenea grado 1

$$\left.\begin{aligned} y &= ux \\ dy &= u\,dx + x\,du \end{aligned}\right\} \begin{aligned} &(ux+x)\,dx = (ux-x)(u\,dx + x\,du) \\ &x(u+1)\,dx = x(u-1)(u\,dx + x\,du) \end{aligned}$$

$$(u+1)\,dx = (u^2-u)\,dx + x(u-1)\,du$$

$$(u+1-u^2+u)\,dx = x(u-1)\,du$$

$$\frac{1}{x}\,dx = \frac{u-1}{-u^2+2u+1}\,du$$

$$\int \frac{1}{x}\,dx = \int \frac{u-1}{-u^2+2u+1}\,du$$

$$\ln x = -\int \frac{-u+1}{-u^2+2u+1}\,du \quad \Bigg\| \quad -\frac{1}{2}\int \frac{-2u+2}{-u^2+2u+1}\,du = -\frac{1}{2}\ln(-u^2+2u+1)$$

$$\ln x = -\frac{1}{2}\ln\left(-u^2+2u+1\right)$$

$$\ln x = -\frac{1}{2}\ln\left(-\frac{y^2}{x^2}+2\frac{y}{x}+1\right)$$

(11) $(x+1)^2 y\,dx + (y-1)^2 x\,dy = 0$

$$(x+1)^2 y\,dx = -(y-1)^2 x\,dy$$

$$\frac{(x+1)^2}{x}dx = -\frac{(y-1)^2}{y}dy$$

$$\int \frac{x^2+2x+1}{x}dx = -\int \frac{y^2-2y+1}{y}dy$$

$$\int\left(x+2+\frac{1}{x}\right)dx = -\int\left(y-2+\frac{1}{y}\right)dy$$

$$\frac{x^2}{2}+2x+\ln x = -\left(\frac{y^2}{2}-2y+\ln y\right)+C$$

$$\frac{x^2}{2}+2x+\ln x+\frac{y^2}{2}-2y+\ln y = C$$

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(15)

$$y' = y^2$$

$$\frac{dy}{dx} = y^2$$

$$dy = y^2 dx$$

$$\frac{1}{y^2}dy = dx$$

$$\int y^{-2}dy = \int dx$$

$$-y^{-1} = x$$

(116)

$$x = -\frac{1}{y}+C$$

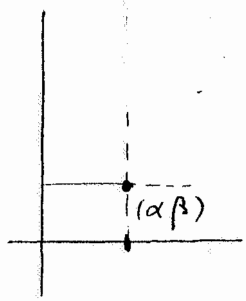
(19)

$$y' = \frac{2x+3y-1}{x-y+2}$$

$$\frac{dy}{dx} = \frac{2x+3y-1}{x-y+2}$$

$$(x-y+2)\,dy = (2x+3y-1)\,dx$$

$$\left.\begin{array}{r} x-y+2=0 \\ 2x+3y-1=0 \end{array}\right\}$$ solución $x=-1$; $y=1$



$$x = X-1$$
$$y = Y+1$$

$$(x-1-(y+1)+2)\;dy = \left[\,2(x-1)+3(y+1)-1\,\right]dx$$

$$(x-1-y-1+2)\,dy = (2x-2+3y+3-1)\,dx$$

$$(x-y)\,dy = (2x+3y)\,dx$$

$$\left.\begin{array}{l} y = ux \\ dy = u\,dx + x\,du \end{array}\right\}$$

$$(x-ux)(u\,dx+x\,du) = (2x+3ux)\,dx$$

$$(1-u)\,u\,dx + (1-u)x\,du = (2+3u)\,dx$$

$$(u-u^2)\,dx + (1-u)x\,du = (2+3u)\,dx$$

$$(u-u^2)-(2+3u)\,dx = -(1-u)\,x\,du$$

$$\frac{1}{x}\,dx = \frac{u-1}{u-u^2-2-3u}\,du$$

$$\frac{1}{x}\,dx = \frac{u-1}{-u^2-2u-2}\,du$$

$$\frac{1}{x}\,dx = \frac{1-u}{u^2+2u+2}\,du$$

$$-\int \frac{2u-2+2-2}{u^2+2u+2}\,du = -\int \frac{2u+2}{u^2+2u+2}\,du + \int \frac{4}{u^2+2u+2}\,du$$

$$= -\ln\left|u^2+2u+2\right| + 4\int \frac{1}{1+(u+1)^2}$$

$$= -\ln\left|u^2+2u+2\right| + 4\,\mathrm{arctg}\,(u+1)$$

$$-\ln\left|u^2+2u+2\right| + 4\,\mathrm{arctg}\,(u+1) = \ln x + \ln C$$

$$4\,\mathrm{arctg}\,(u+1) = \ln c\,x\,(u^2+2u+2)$$

$$4\,\mathrm{arctg}\left(\frac{y}{x}+1\right) = \ln c\,x\left(\frac{y^2}{x^2}+\frac{2y}{x}+2\right)$$

$$4\,\mathrm{arctg}\left[\frac{y-1}{x+1}+1\right] = \ln C(x+1)\left[\frac{(y-1)^2}{(x+1)^2}+\frac{2(y-1)}{x+1}+2\right]$$

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$$\frac{dy}{dx} = \frac{-y+x+1}{2y-2x+3}$$

$$\frac{dy}{dx} = \frac{1+x-y}{3-2x+2y} = \frac{1+(x-y)}{3-2(x-y)} \quad \left\| \begin{array}{l} x-y=t \\ dx-dy=dt \\ dy=dx-dt \end{array} \right\|$$

$$\frac{1+t}{3-2t} = \frac{dx-dt}{dt}$$

$$(3-2t)\,dx-(3-2t)\,dt=(1+t)\,dx$$

$$(2-3t)\,dx=(3-2t)\,dt$$

$$\int dx=\int\frac{3-2t}{2-3t}$$

$$x=\int\left(\frac{2}{3}\,dt-\frac{t}{3}^{-\frac{5}{3}}\right)dt+\int\frac{-3}{2-3t}\,dt$$

$$x=\frac{2t}{3}+\frac{5}{9}\ln|2-3t|-C$$

$$C+x=\frac{2}{3}(x-y)+\frac{5}{9}\ln|2-3x+3y|$$

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$$(3x^2y+2x)\,dx+(x^3-1)\,dy=0$$

$$\frac{\partial P}{\partial y}=3x^2 \quad \frac{\partial Q}{\partial x}=3x^2 \quad \text{Exacta}$$

$$\int(3x^2y+2x)\,dx=x^3y+x^2+C(y)=C$$

$$\cancel{x^3}+C'(y)=\cancel{x^3}-1$$

$$C(y)=-y$$

$$x^3y+x^2-y=C$$