$$\frac{dy}{dx} - \frac{y}{x} = x$$

1°) $\int -\frac{1}{x}\,dx = -\int \frac{1}{x}\,dx = -\ln|x|$

2°) $e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x}$

3°) $\int \frac{1}{x}\cdot x\,dx = \int dx = x$

$$\frac{1}{x}y = x + C$$

$$y = x^2 + xC = x(x+C)$$

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(47) $y' = \frac{4}{x} + x\sqrt{y}$

$y' - \frac{4}{x}y = x\,y^{\frac{1}{2}}$ Bernoulli

$u = y^{1-\frac{1}{2}} = y^{\frac{1}{2}}$ $\quad y = u^2$

$y' = 2u\cdot u'$

$$2u\cdot u' - \frac{4}{x}u^2 = x\,u$$

$u' - \frac{2}{x}u = \frac{x}{2}$ lineal 1er orden en u

1) $\int -\frac{2}{x}\,dx = -2\ln x = \ln x^{-2}$

2) $e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$

3°) $\int \frac{1}{x^2}\cdot\frac{x}{2}\,dx = \frac{1}{2}\ln x$

$$\frac{1}{x^2}\,u = \frac{1}{2}\ln x + C$$

$$\frac{\sqrt{y}}{x^2} = \frac{1}{2}\ln x + C$$

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$$y' + \frac{y}{x} = -xy^2$$

$$y' + \frac{1}{x}y = -xy^2$$ Bernouilli

$$u = y^{1-2} = y^{-1}$$ $$y = \frac{1}{u}$$

$$y' = -\frac{1}{u^2}u'$$

$$\frac{-u'}{u^2} + \frac{1}{x}\cdot\frac{1}{u} = -x\cdot\frac{1}{u^2}$$ multiplicando por $-u^2$

$$u' - \frac{1}{x}u = x$$ lineal 1er orden en u

1) $\int -\frac{1}{x}\,dx = -\ln|x|$

2) $e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x}$

3°) $\int \frac{1}{x}\cdot x\,dx = \int dx = x$

$$\frac{1}{x}y = x + C$$

$$\frac{1}{xy} = x + C$$

$$xy = \frac{1}{x+C}$$

$$y = \frac{1}{x(x+C)}$$

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$$xy' + y - e^x = 0$$

$$y' + \frac{1}{x}y = \frac{e^x}{x}$$

1°) $\int \frac{1}{x}\,dx = \ln|x|$

2°) $e^{\ln x} = x$

3°) $\int + \frac{e^x}{x}\cdot x\,dx = \int + e^x = +\int e^x = +e^x$

$$xy = e^x + C$$

$$\boxed{y = \frac{e^x}{x} + C}$$

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50 $y' - \frac{y}{1-x^2} - 1 - x = 0$

$y' - \frac{1}{1-x^2}\,y = x + 1$ lineal 1er orden

1°) $\int -\frac{1}{1-x^2}\,dx = \int \frac{1}{1+x^2}\,dx = \operatorname{arc\,tg} x$

2°) $e^{\operatorname{arc\,tg} x}$

3°) $\int e^{\operatorname{arc\,tg} x}(x+1)\,dx$ $\left\| \begin{array}{ll} u = x+1 & du = dx \\ dv = e^{\operatorname{arc\,tg} x} & v = \frac{e^{\operatorname{arc\,tg} x}}{1+x^2} \end{array} \right\|$

$I = \frac{(1+x)\,e^{\operatorname{arc\,tg} x}}{1+x^2} - \int \frac{e^{\operatorname{arc\,tg} x}}{1+x^2} = \frac{(1+x)\,e^{\operatorname{arc\,tg} x}}{1+x^2} - e^{\operatorname{arc\,tg} x}$

$e^{\operatorname{arc\,tg} x}\,y = e^{\operatorname{arc\,tg} x}\left(-1 + \frac{1+x}{1+x^2}\right) + e^k$

$e^{\operatorname{arc\,tg} x}\,y = e^{\operatorname{arc\,tg} x}\left(\frac{1+x}{1+x^2} - 1\right) + e^k$

$y = \left(\frac{1+x}{1+x^2} - 1\right) + e^{(k - \operatorname{arc\,tg} x)}$